# Joint Replenishment of Automated Teller Machines


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## ABSTRACT

This paper contributes to supply chain knowledge by considering cash as an asset with an associated stocking (inventory) cost and applying operations research methodologies to optimize automated teller machine (ATM) replenishments. The purpose of this work includes development of a model of economic inventory management that can be replicated in the joint replenishment of a system of ATMs. In the literature, we identify a joint replenishment inventory control model involving computation of a basic cycle-time policy for a joint replenishment problem that is adapted to ATM replenishment. This inventory control model also is used to identify the appropriate replenishment frequency of each ATM corresponding to the minimum relevant costs. An empirical study applies the joint replenishment model to ATMs operating in the city of Ponta Delgada, Portugal. From this study, we conclude that ATM location is important for replenishment frequency and that conclusions from the model application are sensitive to quantification of the parameters.



***Keywords:*** operations management, inventory control, banking


## INTRODUCTION

The joint replenishment problem applies to a variety of situations. Nilsson et al. (2005) describe actual situations in which a single client orders different articles from a supplier, but several products share the same transportation, or in which an item is produced for shipment in different packages. In commercial applications, the joint replenishment problem is similar to when a client orders numerous items from the same distributor with a fixed cost of ordering that is independent of the number of articles bought as well as a variable ordering cost related to each item.

Automated teller machine (ATM) replenishment involves a unique distributor that supplies cash to several ATMs in various locations, with fixed replenishment costs that are independent of the number of ATMs under replenishment and variable ordering costs related to each machine. Serving ATM networks is a costly task that requires employee time, supervision, and important cash management decisions involving high operating costs (financial, transport, handling, insurance, and so on). As interest rates rise and greater operating efficiencies become paramount to competitive advantage, more banks are focused on driving greater efficiencies in how they manage their cash at ATMs. For a detailed reference on cash management optimization for ATM networks, see Simutis et al. (2007).

The methodology proposed in this paper enables institutions to optimize ATM network replenishments by minimizing three cost components: (1) fixed joint ordering costs, (2) variable costs, and (3) holding costs. Fixed costs occur based on each order and are independent of the number of ATMs in a particular order. If such costs are high, it is advantageous for banks to replenish all ATMs in the network at the same time; if they are low, there is no advantage to joint replenishment. Every time a joint replenishment is prepared, variable ordering costs exist related to the inclusion of a specific ATM in the order. Such costs are primarily determined by dislocation costs to the ATM location. Variable ordering costs increase based on the number of ATMs included in the




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joint replenishment. Finally, holding costs reflect the opportunity costs of maintaining cash in the ATM. Holding costs are high when the number of replenishments is small, given the amount of cash accumulated in the ATM. Further, efficient ATM management may significantly reduce holding costs.

Liquidity management enables companies to achieve the best profitability from underutilized funds and increased control over cash flows. Cash and liquidity management requires forecasting company cash needs and then managing capital in the most efficient manner to ensure that cash needs can be met. Accordingly, efficient ATM management is vital to liquidity management for financial institutions.

Research on ATM management is scarce. The most relevant papers are by Nakamura et al. (2003) and Wang et al. (2003). Nakamura et al. (2003) propose a stochastic model for an inside-branch ATM with two breakdowns that operate unattended on the weekend and are checked after trouble occurs. The authors derive an optimal maintenance policy that minimizes expected ATM costs for an unattended operating period. Wang et al. (2003) discuss p-median, facility location, and flow-capturing models relative to their use in ATM server locations.

In this paper, we present Viswanathan's (1996) algorithm used to calculate the restricted cycle policy for joint replenishment of ATMs in the next section, and present empirical analysis in the section titled "Empirical Application." In the empirical analysis, we apply the joint replenishment problem to all financial institutions with ATMs in the city of Ponta Delgada, Portugal. Table 1 shows all financial institutions operating in the city and the number of ATMs that correspond to each. All institutions have their headquarters outside the region.

Historically, the *BCA-Banco Comercial dos Açores* (In the table, *BANIF Açores*) was the primary financial institution in the region. For more than a century, BCA was a public institution owned by the Azores regional government. As such, it was the only bank with its headquarters in Azores and is deeply ingrained in the regional economy.

In 1996, the Azores regional government split its stake of the bank with the public and subsequently converted to a private financial institution called BANIF-*Banco Internacional do Funchal*, which acquired 56 percent interest in BCA. In 2007, all BCA branches were renamed BANIF and became branches of the private institution. These new BANIF branches have retained most of BCA's market and traditions and still dominate the Azores financial industry, with branches in almost all villages in the region. Nevertheless, other banks with a lesser presence are increasingly investing in the region, as a number of new branches and ATMs have opened, primarily in the city of Ponta Delgada.

**TABLE 1**: Financial Institutions

| Financial Institution | Number of ATM Machines in the City of Ponta Delgada |
|---|---|
| **BANIF Açores** - *Banco Internacional do Funchal* | 18 |
| **BPI** – *Banco Português de Investimentos* | 4 |
| **BTA** - *Banco Totta Açores* | 5 |
| **BESA**- *Banco Espírito Santo dos Açores* | 5 |
| **GCAM**- *Caixa de Crédito Agrícola Mútuo dos Açores* | 2 |
| **CGD**- *Caixa Geral de Depósitos* | 4 |
| **Millennium-BCP**- *Banco Comercial Português* | 5 |
| **MONTEPIO** | 5 |



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## THE JOINT REPLENISHMENT PROBLEM AND ATM REPLENISHMENT

Algorithms to solve the joint replenishment problem show progressive improvement over the last few decades. Goyal (1974) integrates the restricted cycle policy (that is, the cyclical stock revision policy) so that at least one article must be replenished each time a joint order is placed. Goyal was the first to propose an algorithm to calculate the minimum point of the joint replenishment total cost function that identifies a lower and upper bound for the cycle time, between which one of the values is optimal.

Andres et al. (1975) noted an inaccuracy in the development of Goyal's (1974) algorithm relative to calculation of the lower limit. Later, Goyal (1988) agreed that his algorithm did not guarantee achievement of the best solution, while Van Eijs (1993) affirmed that Goyal's algorithm excluded the choice of a global-cycle policy. The authors suggested a slight adjustment that makes it possible to obtain the optimal cyclical strategy.

Viswanathan (1996) presented an algorithm based on developments of Goyal (1974) and Van Eijs (1993), proposing identification of two tighter limits for cycle time that results in less computational effort to reach the best solution. More recently, Fung et al. (2001) combined knowledge from Goyal (1974) with the alterations introduced by Andres et al. (1976) and Van Eijs (1993) to present a new algorithm for solution of a model that involved both cycle policies: restricted and global.

Viswanathan (2002) argued that the methodology followed by Fung et al. (2001) to identify restricted cycle policy does not guarantee the best solution. In an empirical study, Viswanathan concluded that his algorithm required less computational time and less iteration than other known algorithms. Accordingly, we use Viswanathan's algorithm to solve the joint replenishment problem that corresponds to the ATM replenishment concerned here.

In the present study, we briefly describe the joint replenishment problem. For a complete review of the problem, refer to Khouja and Goyal (2008).

### Terminology

We adopt the following terminology for the joint replenishment problem:

$n$ – Number of ATMs

$A$ – Fixed cost of joint ordering, independent of the number of ATMs (major setup cost).

$a_i$ – Variable cost of including ATM $i$ in joint replenishment; $i = 1, 2, \dots, n$

$h_i$ – Holding cost (maintenance) of one unit of cash in the ATM by unit of time

$d_i$ – Demand for cash at ATM $i$ by unit of time; constant and known

$T$ – Joint replenishment cycle time; this is, the period of time that elapses between each revision of stocks (*basic cycle time*)

$t_i$ – Period of time that elapses between each replenishment of ATM $i$; $i = 2, \dots, n$

$V$ – Mean number of joint replenishment orders by unit of time

$v_i$ – Mean number of replenishment orders for ATM $i$ by unit of time ($vi = 1/ti$), with $i = 1, 2, \dots, n$

$k_i$ – Frequency of replenishment of ATM $i$, assuming discrete values that are multiples of T

$K$ – Vector composed of the replenishment frequency for each ATM ($k_1, k_2, \dots, k_n$)

$C_{tr}$ – Relevant average total costs of the joint replenishment system by unit of time

### Assumptions

The following assumptions are often made in the literature regarding the joint replenishment problem with continuous revision and deterministic demand:

- Demand is known and constant.
- Stock replenishment admits noninteger quantities of the items (principle of the divisibility of the variables).
- The costs of output and/or the prices of acquisition do not depend on quantities ordered (no quantity discounts).
- Stock replenishment is immediate, assuming an infinite availability quantity for each item.
- Stock shortage is not admitted.
- The waiting time of supply is zero.
- There is no limitation of storage space.
- Operation of the storage system admits an infinite time horizon.



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- The joint replenishment of orders requires that at least one of the items is always ordered, (that is, to have T as periodicity of the order, or "restricted cycle policy").

These assumptions require that if deterministic demand exists without the possibility of stock shortage and with instant replacement, it makes sense to order an item only when inventory falls to the zero level. It is notable that these assumptions exclude other joint replenishment forms. Relevant costs associated with the problem of joint replenishment in accordance with the model assumptions are classified as ordering and holding costs. Ordering costs are subdivided into two components: a fixed component A (major setup cost), which is incurred whenever an order occurs, independent of the number of ATMs involved in the replenishment; and a variable cost component ($a_i$) that is related to each item integrated in the order (minor setup cost). Every time a joint replenishment is prepared, a variable ordering cost exists that is related to the inclusion of a specific ATM in the order.

### Variable Ordering Costs

Variable ordering costs relate to launching orders, or the preparation and start of a production order, and include the administrative costs of service purchases—namely, the communications costs and time for decisions as well as the qualitative and quantitative reception of orders. In the case of ATM replenishment, these expenses relate to dislocation for cash and materials replenishment, the automatic detection of differences, the registering of faults and excesses, and other operations that are indispensable to the process. As other costs assume nonexpressive small values, the primary expenses we consider to be relevant in the replenishment of an ATM correspond to the respective dislocation costs. We assume that such costs are proportional to the distance from the ATM location to distribution headquarters. Such measurement is considered an approximate dislocation time when using a vehicle to travel access roads in the direction permitted by traffic rules.

Banks that opt to hire external services to supply ATMs in large urban centers in Portugal, such as Lisbon or Oporto, assume a cost of approximately 30 euros for each visit to the ATM. In the case under study, we also associate a value of 30 euros to each ATM, attributing a standard cost of 5 euros and adding 25 euros for every two additional kilometers.

### Fixed Cost of Joint Replenishment

Fixed joint replenishment costs are an element of total costs that are difficult to quantify. In an industrial situation, such costs represent the administrative expenses incurred when launching a production order (Goyal 1974). If these costs are very low, there is no significant advantage to establishing joint replenishment; on the other hand, if such costs are high, it is more advantageous to replenish all network ATMs at the same time.

This component of the model corresponds to the costs a banking institution attributes to preparing an exit for the joint replenishment of ATMs, independent of the number of points it visits. We include the costs associated with the support and analysis of the electronic information of the stock evolution in ATMs, human resources and transportation costs, as well as the costs of obtaining cash from the Delegation of the Bank of Portugal.

In the empirical work studied here, we consider a value of 100 euros to be the fixed cost of replenishment $A$ in the first analysis. In our second analysis, we consider a simulation of 1,000 different values for $A$ generated via uniform distribution, varying between 100 euros and 1,000 euros.

### Holding Costs

Holding costs by unit of time, $h_i$, result from the maintenance of each unit of cash in ATM $i$ while waiting for its use. The monthly holding cost of cash in an ATM can be inferred through its capital opportunity cost as well as via inclusion of other direct monetary costs: equipment systems for handling information, maintenance, repair, security, and energy consumption costs.

For simplicity, we consider holding costs to be only the capital opportunity cost of each banking institution involved in the management



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of ATMs. We analyze the Reports and Consolidated Accounts of the financial groups that integrate banks that support ATMs as of December 31, 2004.

To calculate capital opportunity cost, we consider values in the income statement as financial returns (in the equation below, FR), including interest and compared profits, asset returns, service commissions and profits obtained as a result of operations. As active investments (AI), we consider accounts in the balance sheet related to credits to other financial institutions, loans to clients, asset investments, and other financial activities such as participation in associated company activities. The holding costs $h_i$ (capital opportunity cost) for each financial institution $i$ can then be calculated as.

$$h_i = \frac{FR_i}{AI_i}$$

Table 2 displays the holding costs for each 1,000 euros immobilized in ATMs for the several financial institutions over the time period under study. As one can see from the table, some institutions have higher levels of capital opportunity cost, as are the cases of BES (Banco Espírito Santo) and CGD (Caixa Geral de Depósitos). This means these institutions have been more successful generating returns from investments. The practical consequences for our joint replenishment problem are that not all ATMs will have the same holding costs; those costs will vary according to the ATM ownership, as ATMs belonging to institutions with higher COC will have higher holding costs.

### Objective

The primary objective of joint replenishment is to find a balance between the fixed ordering costs and holding costs of various items through adjustment in the frequency of replenishments ($k_i$) for each item (in our case, each ATM). The problem involves the calculation of a base time period ($T$) that corresponds with the joint replenishment policy cycle and its multiple positive integers ($k_i$'s) for the frequency of each ATM replenishment. Thus, the objective is to find the values of $T$ and $k_i$ that drive relevant total costs for the period of time ($C_{tr}$) to the minimum value.

The time interval $t_i$ is calculated by $k_i.T$. In the same way, $v_i$ can be calculated from $V/ki$, assuming that $V = 1/T$.

In a deterministic model, the quantity of a unique item to order corresponds with the expression $Q = d.T$, where $d$ is demand by unit of time. In a joint replenishment model, the quantity of each item to order can be expressed as $q_i = d_i.T$. But as item i is to be jointly replenished, we weight the quantity by its replenishment frequency as a function of $T$. Therefore, we can identify the quantity to order from the expression $q_i = k_i.d_i.T$.

Since inventory reaches zero at the end of the time period, average stock can be calculated from $(q_i + 0)/2$, which corresponds to $q_i/2$ or $(k_i.d_i.T)/2$. The holding cost of this average inventory for the time period is calculated using the respective unitary cost ($h_i$). Therefore, it is equal to $(h_i.k_i.d_i.T)/2$.

On the other hand, ordering cost can be identified in the following way: For ATM $i$, the cost of ordering will be $a_i$, but as orders are conducted with frequency $k_i$, the cost incurred when launching the order for ATM $i$ is $a_i/k_i$.

Grouping all ATMs that are the object of a joint replenishment, we identify the medium relevant total costs by unit of time equation:

$$C_{tr}(T,K) = \frac{A + \sum_{i=1}^{n} \frac{a_i}{k_i}}{T} + \sum_{i=1}^{n} \frac{h_i.d_i.k_i.T}{2} \qquad (1)$$

We want to minimize this function subject to the constraints of $T>0$ and $k_i \geq 1$ and integer. The algorithm used to solve the problem is adapted from Viswanthan (1996) and is described in the appendix of our paper.

**TABLE 2**: Monthly Holding Cost (for 1.000€)

| Financial Institution | BANIF | BPI | BTA | BESA | GCAM | CGD | BCP | MONTEPIO |
|---|---|---|---|---|---|---|---|---|
| $h_i$ | 4.96 | 7.62 | 5.64 | 10.78 | 5.75 | 10.13 | 6.17 | 4.44 |



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**TABLE 3**: The Best Solutions

| Month | Optimal Solution | | | | Processing Time |
|---|---|---|---|---|---|
| | Ctr* (in euros) | T* (months) | T* (days) | N.° Iterations | |
| Jan-05 | 9068.4 | 0.1513 | 4.5 | 18 | 0.093 |
| Feb-05 | 9055.1 | 0.1498 | 4.5 | 3 | 0.016 |
| Mar-05 | 9229.4 | 0.1464 | 4.4 | 24 | 0.110 |
| Apr-05 | 9198.1 | 0.1468 | 4.4 | 21 | 0.109 |
| May-05 | 9289.2 | 0.1470 | 4.4 | 3 | 0.015 |
| June-05 | 9240.0 | 0.1436 | 4.3 | 21 | 0.094 |
| July-05 | 9384.3 | 0.1435 | 4.3 | 1 | 0.015 |
| Aug-05 | 9384.8 | 0.1449 | 4.3 | 1 | 0.015 |
| Sep-05 | 9293.7 | 0.1453 | 4.4 | 8 | 0.047 |
| Oct-05 | 9350.5 | 0.1448 | 4.3 | 1 | 0.016 |
| Nov-05 | 9294.3 | 0.1452 | 4.4 | 6 | 0.015 |
| Dec-05 | 9304.6 | 0.1450 | 4.4 | 1 | 0.031 |

## EMPIRICAL APPLICATION

The present empirical analysis is based on data collected by *Sociedade Interbancaria de Serviços* (SIBS), the Inter-Bank Service System of Portugal for the city of Ponta Delgada and bordering zones. The system includes 48 active ATMs with at least two years of activity under the management of eight credit institutions. SIBS operates *Multibanco*, an interbank network in Portugal that in 2008 linked the ATMs of 27 banks with more then 13,000 machines. *Multibanco* was created in 1985. In the beginning, only cash withdrawals and balance and checking transaction statements were available. Later, it introduced features such as service and shopping payments, government payments, cell phone prepaid card loading, check requisitions, and other offerings. By 2008, more than 60 different features were available via ATMs in Portugal.

*Multibanco* is a fully integrated interbank network that offers standardized service. ATM features are the same for all machines, and cash capacities are standardized to 5,000 bills; half of the capacity is available for 5 euro bills and the other half for 10 euro bills (a maximum of 75,000 euros). Nevertheless, not all ATMs require the same amount of service. In 2005, on average, total withdrawals for one month from one machine in the city of Ponta Delgada were 133,541 euros with a standard deviation of 77,077 euros.

We collected monthly demand for cash data for each ATM over the 1999-2004 period to predict demand for cash from the 48 ATMs using time-series analysis. Using a PC Pentium® with 504 megabytes, we programmed the algorithm in C++ language to analyze how to support ATMs.

We start by computing the best solutions for joint ATM replenishment for each of the 12 months of 2005. Table 3 shows the best solutions for each month we obtained values for, relevant to total costs, cycle time, number of possible solutions enumerated, and respective processing times. The model arrives at the best solutions coincidental with Phase C of the algorithm described in the appendix, using an average of eight iterations and requiring an average of .043 of a second. It is notable that in the months of July, August, October and December, it was possible to solve the problem using only one iteration.

Analyzing the solutions, we see that joint ATM replenishment is optimized via a cycle time oscillating between four and five days; an order of joint replenishment must be emitted in a time interval of four to five calendar days. Cycle time is shorter during the summer, which reflects greater demand for cash during these months.

The relevant costs of a joint replenishment policy for ATMs reaches, on average, 9,300 monthly euros, varying from 9,055.1 euros in February to 9,384.3 euros in July and August. The model



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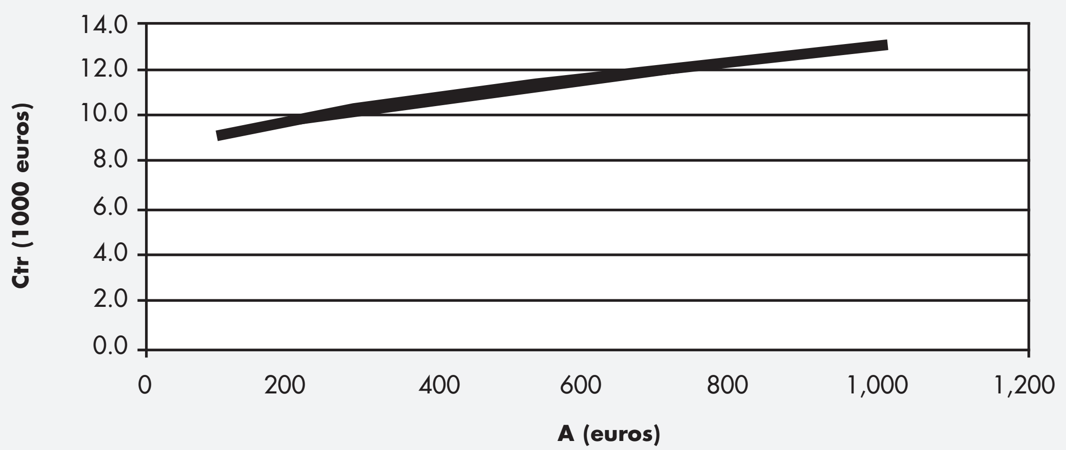
**TABLE 4**: Optimal Joint Replenishment

| ki* = 1; ti* = 3 days | qi* | ki* = 2; ti* = 6 days | qi* | ki* = 3; ti* = 9 days | qi* |
|---|---|---|---|---|---|
| M01, M15 | 10 | M08 | 47 | M04 | 22 |
| M02 | 16 | M11 | 32 | M12 | 31 |
| M03, M17 e M31 | 14 | M14 | 8 | M16 | 15 |
| M05 | 21 | M20 | 21 | M27 | 28 |
| M06, M07 | 13 | M25 | 18 | M43 | 24 |
| M09 | 27 | M30 | 27 | M45 | 30 |
| M10 | 23 | M34 | 31 | M47 | 11 |
| M13 | 26 | M35 | 11 | **ki* = 4; ti* = 12 days** | **qi*** |
| M23 | 6 | M37 | 20 | M18 | 16 |
| M24, M48 | 8 | M38 | 22 | M21 | 42 |
| M28, M29, M32 | 19 | M39 | 41 | M22 | 39 |
| M33 | 18 | M40 | 50 | **ki* = 5; ti* = 15 days** | **qi*** |
| M36 | 5 | M41 | 58 | M19 | 16 |
| M42 | 20 | M46 | 34 | M26 | 14 |
| M44 | 7 | Unit: €1000 | | | |

**FIGURE 1**: Ctr Behavior




also provides information on the frequency with which each ATM should be replenished, in reference to period $T^*$.

The quantities of cash inventory require consideration for the predicted demand of each ATM. Table 4 transcribes the optimal joint replenishment plan. While most ATMs are replenished every three, six or nine days, some require replenishment only every 12-15 days. As expected, ATMs with more frequent replenishment are located in the historic center of the city, which has the highest traffic rates.

Variable costs required to supply each machine are proportional to the distance traversed. When demand is low, the average cash inventory immobilized is also low. Therefore, the costs incurred within a period of immobilized capital are more than compensated by dislocation cost savings.

As a second exercise, we simulate the values of parameter *A*, obtaining the solution for 1,000 different problems and reporting the best total cost ($Ctr^*$), cycle time ($T^*$), and frequency of supply-



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ing each ATM ($k_i^*$). Since the fixed costs of joint ordering are difficult to quantify, we developed the present simulation exercise to measure the impact in total costs and cycle time of fixed cost variations.

Total costs vary between a minimum of 9,319 euros and a maximum of 13,011 euros, with an average of 11,372 euros and a standard deviation of 1,058 euros. The behavior of $C_{tr}$ presents a directly proportional growth tendency to the evolution of $A$ while its extreme values coincide with the lower and higher simulated values for $A$ (See Figure 1).

The best solutions calculate an average cycle time of 0.2518 months, with a standard deviation of 0.0437 months (approximately one day), registering a minimum and maximum of 0.1451 months (approximately four days) and 0.3154 months (approximately nine days). These numbers imply that in any empirical study, some attention must be taken to the values of parameters.

## CONCLUSIONS

This work identifies one way to optimize joint ATM replenishment using theoretical models from the operations research field. We confirm the hypothesis that a model for the joint replenishment of inventory, frequently used in commercial and industrial frameworks, is useful to a cash management system related to efficient ATM management. We also confirm that it is possible to identify and quantify all of the model parameters. Using Viswanathan's (1996) algorithm, we identify the best solution for joint replenishment.

The empirical study, conducted in 2005, aimed to show that there are advantages in the joint replenishment of all ATMs belonging to all banks operating in the city of Ponta Delgada. Nevertheless, to date, all banks in Ponta Delgada continue to manage their ATMs independently, probably due to the relatively small number of ATMs operated by each bank in this city.

In Portugal's largest cities, Lisbon and Oporto, ATM management is fully outsourced. Full outsourcing in the region presently includes all operations related to ATM management, including deployment, cash replenishment, maintenance, monitoring, and armored car support. Our model is particularly useful for firms offering such services. In the city of Ponta Delgada, it is not yet possible for financial institutions to fully outsource ATM management and, thus, banks in the region are unable to capitalize on the advantage of joint ATM replenishment by a single firm. But full outsourcing is a particularly compelling advantage for small- to mid-sized financial institutions, such as the kind presently operating in Ponta Delgada. Our paper suggests that a unique opportunity exists for replenishment firms to offer this service to the city's financial institutions.

Despite the high costs of ATM management, ATM networks are proven to help attract new customers and promote lower-cost, self-service banking. Both for financial institutions managing their own ATMs and firms offering this service, joint replenishment models are a useful tool for banks to realize operational efficiencies related to cash replenishment.

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## APPENDIX

### Viswanathan's Algorithm

The algorithm involves the following steps:

**Part A: Calculation of the interval for the cycle time: [$T_{min}$; $T_{max}$]**

1. Compute:

$$T_{max} = \sqrt{\frac{2\left(A + \sum_{i=1}^{n} a_i\right)}{\sum_{i=1}^{n} h_i.d_i}} \quad \text{and} \quad T_{min} = \min_{1 \le i \le n} \sqrt{\frac{a_i}{h_i.d_i}}$$

2. Compute $K(T_{min})$, using the inequality

$$k_i(T_{min})[k_i(T_{min})-1] < \frac{2a_i}{h_i.d_i.T_{min}^2} \le k_i(T_{min})[k_i(T_{min})+1]$$

and compute the respective total relevant cost

$$C_{tr}(T_{min}, K) = \frac{A + \sum_{i=1}^{n} \frac{a_i}{k_i}}{T_{min}} + \sum_{i=1} \frac{h_i.d_i.k_i.T_{min}}{2}$$

3. Replace $T_{min}$ using the following result

$$T_{min} = \max\left[\min_i \sqrt{\frac{a_i}{h_i.d_i}}; \frac{2A}{C_{tr}(T_{min}, K)}\right]$$

**Part B: Improvement of the interval for cycle time: [$T_A$ ; $T_B$]**

$B_1$ – Improving the $T_{min}$

4. Assign a value to the ratio *Min-Imp*.

If $T_{max} / T_{min} \le$ Min-Imp, go to step 9, doing $T_A = T_{min}$, otherwise, go to step 5.

5. Take $T_O = T_{min}$

6. Compute $K(T_O)$ from inequality (3)

Compute $T_A$ using:

$$T[K(T_O)] = \sqrt{\frac{2\left(A + \sum_{i=1}^{n} \frac{a_i}{k_i}\right)}{\sum_{i=1}^{n} h_i.d_i.k_i}} \qquad (5)$$

7. If $(T_A / T_O) >$ *Min–Imp*, go back to step 4.

8. If $T_A < T_{min}$, take $T_A = T_{min}$ given in step 1 of Part A of the algorithm; otherwise take $T_A = T_O$.

$B_2$ – Improve $T_{max}$

9. If $T_{max} / T_A \le$ Min-Imp, go to step 13 taking $T_B = T_{max}$; otherwise, go to step 10.

10. Take $T_O = T_{max}$

11. Compute $K(T_O)$ using inequality (3)

Compute $T_B$ using the value $T[K(T_O)]$ from equation (5)

12. If $(T_O / T_B) >$ *Min-Imp*, go to step (9); otherwise, continue.



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### Part C: Computing the optimal solution

13. Identify all of the subintervals of $T$ between $T_A$ and $T_B$ corresponding to the computation of given constant vector $K$:

$$\sqrt{\frac{2a_i / h_i.d_i}{k_i(T).\left[k_i(T)+1\right]}} \leq T < \sqrt{\frac{2a_i / h_i.d_i}{k_i(T).\left[k_i(T)-1\right]}}$$

14. For each constant vector $K$, compute the respective total cost:

$$C_{tr}(K) = \sqrt{2\left(A + \sum_{i=1}^{n} \frac{a_i}{k_i}\right) \times \sum_{i=1}^{n} h_i.d_i.k_i}$$

Accept $C_{tr}^{*}(K^{*}) = \min C_{tr}(K)$ as the optimal solution and compute $T^{*}$.



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